2. Logarithms

- If a is any positive real number (except 1), n is any rational number and $a^n = b$, then n is called the **logarithm** of b to the base a, and is written as $\log_a b$.

  Thus, $a^n = b$ if and only if $\log_a b = n$.

  $a^n = b$ is called the exponential form and $\log_a b = n$ is called the logarithmic form.

- **Properties of logarithm:**
  - Logarithms are only defined for positive real numbers.
  - $\log_a 1 = 0$ and $\log_a a = 1$ where, a is any positive real number except 1.
  - $\log_a x = \log_a y = n(\text{say}) \Rightarrow x = y$
  - Logarithms to the base 10 are called common logarithms.
  - If no base is given, the base is always taken as 10. For example, log 5 = $\log_{10} 5$

- **Laws of logarithm:**
  - **Product Law:**

    $\log_a mn = \log_a m + \log_a n$

    In general, $\log_a(mnp...) = \log_a m + \log_a n + \log_a p + ...$

  - **Quotient Law:**

    $\log_a \frac{m}{n} = \log_a m - \log_a n$

  - **Power Law:**

    $\log_a m^n = n \log_a m$

Example:

Find the value of x if $\log_7 343 = 5x - 4$.

Solution:

$\log_7 343 = 5x - 4$

$\Rightarrow 7^{(5x-4)} = 343$

$\Rightarrow 7^{(5x-4)} = 7^3$

$\Rightarrow 5x - 4 = 3$

$\Rightarrow 5x = 7$

$\Rightarrow x = \frac{7}{5}$

For any positive number N, $\log_{10} N$ = Characteristic + Mantissa.

Characteristic of Logarithm

The characteristic of the logarithm of a positive number $N \geq 1$ is positive.
If a positive number $N \geq 1$ has m digits in its integral part, then subtract 1 from m to find the characteristic of the logarithm—that is, characteristic $\log_{10} N = m - 1$.
The characteristic of the logarithm of a positive number $0 < N < 1$ is negative.
If there are k zeroes immediately after the decimal point, then the characteristic of the logarithm is $\log_{10} N = -(k + 1)$.

Mantissa of Logarithm

Mantissa of the logarithm of a one-digit number:

Consider the number 4. The mantissa of log 4 will be the value of 20 under 0, i.e., 0.6021.

Mantissa of the logarithm of a two-digit number:

Consider the number 42. The mantissa of log 42 will be the value of 42 under 0, i.e., 0.6232.

Mantissa of the logarithm of a three-digit number:

Consider the number 423. The mantissa of log 423 will be the value of 42 under 3, i.e., 0.6263.

Mantissa of the logarithm of a four-digit number:

Consider the number 4235. The mantissa of log 4235 will be the value of 42 under 3 + mean difference under 5, i.e., 0.6268 = (6263 + 5).

Mantissa is always positive.

The decimal point is ignored while finding the mantissa of the logarithm of a number.

Antilog

If $\log_{10} 4.235 = 0.6268$, then antilog 0.6268 = 4.235.

Method to find the antilogarithm of a number:

1. Identify the characteristic p and mantissa q of the logarithm.

2. The antilog of mantissa can be found using the antilog table.

3. If the characteristic p is positive, then place the decimal after $(p + 1)$ digits from the left; and if the characteristic p is negative, then place $(p - 1)$ zeroes before the first digit.

Antilog tables are used to find the antilog of the decimal part.